The Effects of Single-Player Coalitions on Reward Divisions in Cooperative Games

Greg d'Eon, Kate Larson, Edith Law

University of Waterloo

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This Talk

Research question: how do people pick fair reward divisions when acting as impartial decision makers?

Explore how values of single-player coalitions affect these divisions

Show that rewards are often unrelated to Shapley value: people break null player and additivity axioms

Cooperative Games

A *transferable utility game* describes how a group of players can earn rewards by working together in coalitions

Players	**Reward**
(nobody)	0
Alice	30
Bob	10
Charlie	0
Alice, Bob	60
Alice, Charlie	30
Bob, Charlie	10
Alice, Bob, Charlie	60

How to fairly divide the reward among them?

The Shapley Value

Shapley value [Shapley 1953]:

- Consider all possible orders of players joining the group
- Give players their *average marginal contribution* over these orders

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Unique reward division satisfying 4 fairness axioms

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2. **Symmetry**: players with *same marginal contributions* to all coalitions get same reward

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2. **Symmetry**: players with *same marginal contributions* to all coalitions get same reward
3. **Null Players**: players with *no marginal contribution* to any coalition get no reward
4. **Additivity**: for all games f and g, $Sh(f+g) = Sh(f) + Sh(g)$

Alternative Values

Are these axioms *fair*?

Alternative values:

- Solidarity value [Nowak and Radzik 1994]
- Egalitarian Shapley values [Joosten 1996, Casajus and Huettner 2013]
- Procedural values [Malawski 2013, Radzik and Driessen 2013]

All three weaken null player axiom

Empirical Studies

Prior work: empirical studies of cooperative games

Most focus on bargaining [Kalisch et al. 1954, Kahan and Rapoport 1984, Maschler 1992]

Impartial decisions about reward divisions [De Clippel et al. 2013]

- Rewards are convex combinations of equal split and Shapley value
- Rewards satisfy efficiency, symmetry, and additivity, but not null player
- Limitation: only studies *zero-normalized games*

Experiments

Question: How do single-player coalitions affect people's impartial reward divisions?

Answer this question through two experiments

- **Experiment 1:** Do people put more weight on 1- or 2-player coalitions' values?
- **Experiment 2:** How do people reason about 1-player coalitions?

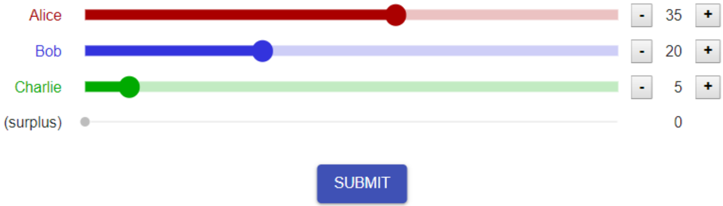
Experiment Interface

Experiment: divide rewards in fictional scenario

Players	Gold Pieces
(nobody)	0
Alice	30
Bob	20
Charlie	10
Alice, Bob	50
Alice, Charlie	40
Bob, Charlie	30
Alice, Bob, Charlie	60

All three of them go on the quest together and earn **60** gold pieces as a group.

How should they divide the gold?



Procedure

Within-subjects experiments

- Participants selected rewards for 11 or 17 games
- Hired 100 workers from Mechanical Turk for each experiment

Filtered out low-quality workers

- Spending under 5 seconds on any screen
- Submitting blatantly non-sensical answers

Experiment 1

Experiment 1: designed games to emphasize values of 1- or 2-player coalitions

	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3

Experiment 1

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- Choose target Shapley value

	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3
									25	25	10

Experiment 1

Experiment 1: designed games to emphasize values of 1- or 2-player coalitions

- Choose target Shapley value
- Design game where only 1-player values differ

	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3
Solo	0	40	40	10	60	60	60	60	25	25	10

Experiment 1

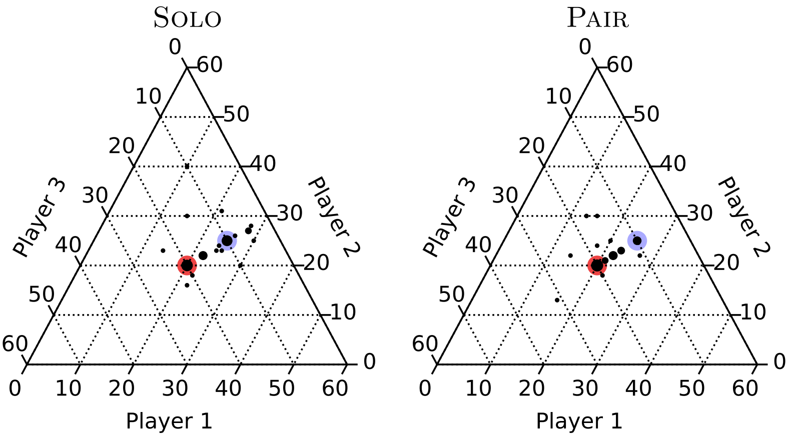
Experiment 1: designed games to emphasize values of 1- or 2-player coalitions

- Choose target Shapley value
- Design game where only 1-player values differ
- Design game where only 2-player values differ

	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3
Solo	0	40	40	10	60	60	60	60	25	25	10
Pair	0	0	0	0	45	15	15	60			

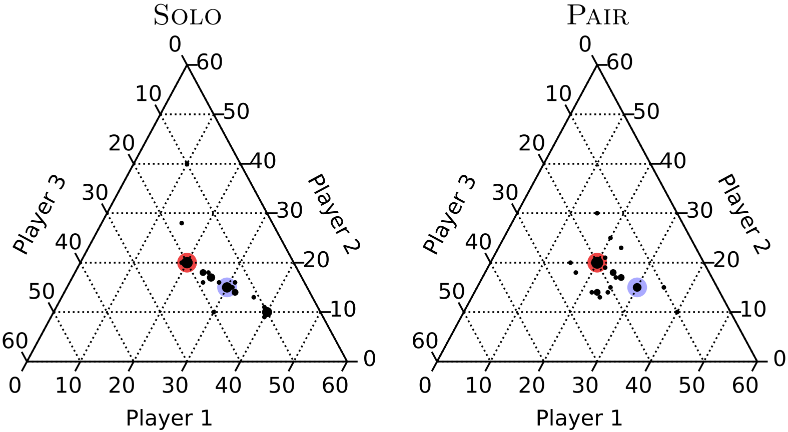
Experiment 1

Shapley value $= [25, 25, 10]$ (1-Worse)



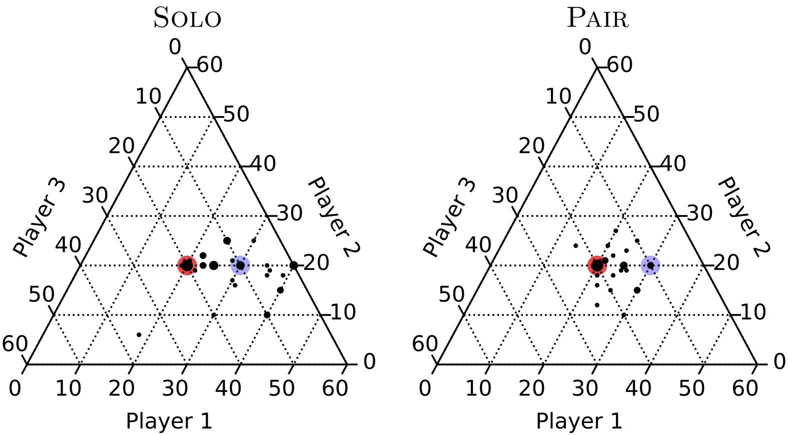
Experiment 1

Shapley value $= [30, 15, 15]$ (1-BETTER)



Experiment 1

Shapley value $= [30, 20, 10]$ (Distinct)



Experiment 2

Experiment 1: 1-player coalition values have larger effect on people's reward divisions

Goal of Experiment 2: understand how people reason about these values

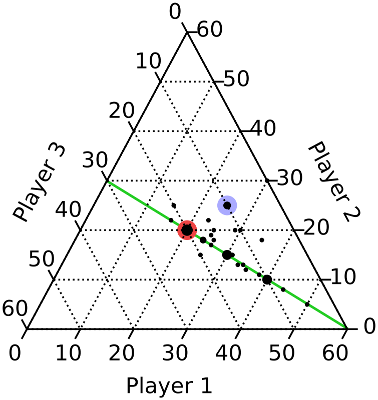
Focus on three features:

- 1-player values not a multiple of the Shapley value
- Varying sum of 1-player values
- Games with null players

Experiment 2

Shapley value $= [25, 25, 10]$, with 1-player values $[20, 5, 5]$:

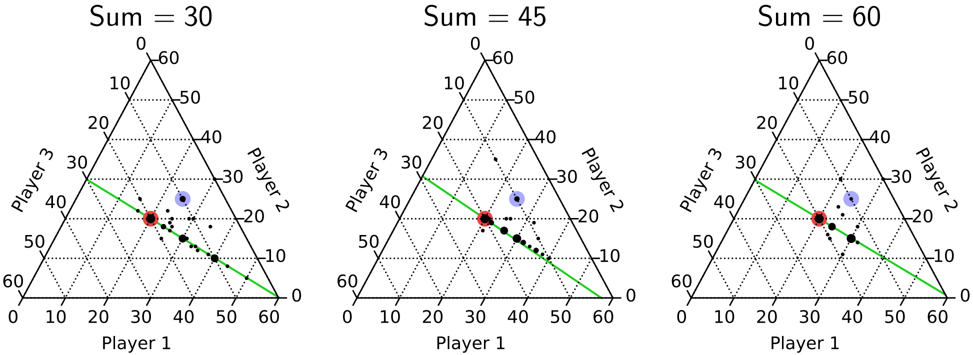
Game								Shapley value		
$\emptyset$	1	2	3	12	13	23	123	1	2	3
0	20	5	5	60	30	45	60	25	25	10



Experiment 2

Shapley value $= [25, 25, 10]$, with 1-player values summing to 30, 45, or 60:

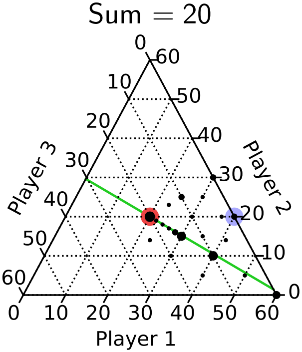
	Game								Shapley value		
Sum	$\emptyset$	1	2	3	12	13	23	123	1	2	3
30	0	20	5	5	60	30	45	60	25	25	10
45		25	10	10							
60		30	15	15							

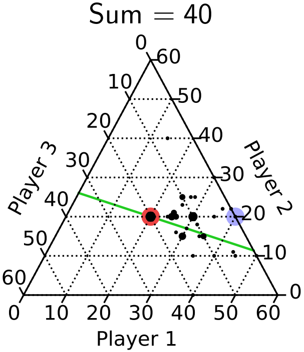


Experiment 2

Shapley value $= [40, 20, 0]$, with player 3 null

	Game								Shapley value		
Sum	$\emptyset$	1	2	3	12	13	23	123	1	2	3
20	0	20	0	0	60	20	0	60	40	20	0
40	0	30	10	0	60	30	10	60			





Testing Axioms

Experiment 2: reward divisions are quite consistent, but unrelated to the Shapley value

Which axioms did people violate?

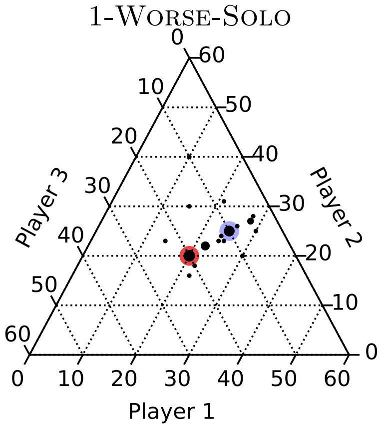
- Efficiency was required by experiment interface
- Use statistical tests to check symmetry, null player, and additivity

Testing Axioms: Symmetry

To satisfy symmetry, must give equal rewards to symmetric players

- Experiment 1 games had symmetric players
- Most people gave equal rewards – no significant differences

Symmetry: ✓



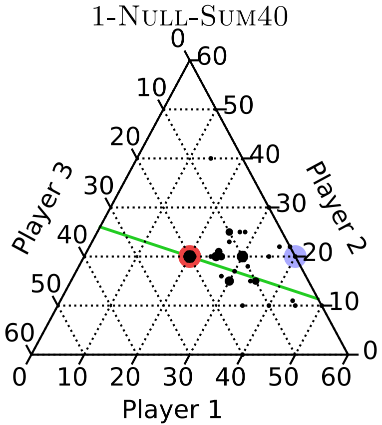
Testing Axioms: Null Player

To satisfy null player axiom, must give no reward to null players

- 4 games in Experiment 2 with null players
- Best case: 14 of 74 participants gave 0 reward

Null player: ✗

- Consistent with De Clippel [De Clippel et al. 2013]



Testing Axioms: Additivity

To test additivity, need to know relationship between two games

Games from Experiment 2:

	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3

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f	0	20	5	5	60	30	45	60	25	25	10

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	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3
f	0	20	5	5	60	30	45	60	25	25	10
g		25	10	10							

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	Game								Shapley value		
Condition	$\emptyset$	1	2	3	12	13	23	123	1	2	3
f	0	20	5	5	60	30	45	60	25	25	10
g		25	10	10							
$g - f$	0	5	5	5	0	0	0	0			

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f	0	20	5	5	60	30	45	60	25	25	10
g		25	10	10							
$g - f$	0	5	5	5	0	0	0	0	0	0	0

To satisfy additivity, must give same rewards for these games

Testing Axioms: Additivity

Found that people gave inconsistent rewards to players 1 and 3

- Significant in 1-WORSE games ($p < 0.01$)
- Marginally significant in 1-BETTER games ($p = 0.07$ and $p = 0.08$)

Additivity: ✗

- Conflicts with [De Clippel et al. 2013]

Describing Human Reward Divisions

Models for people's reward divisions?

- Had little success fitting procedural values
- Heuristics similar to equal division payoff bounds [Selten 1987]
- Shapley value after applying non-linear utility function to coalition values
- Shapley value with weaker additivity axiom
- Stability concerns

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